The Mechanics of
Tractor - Implement Performance

Theory and Worked Examples

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CHAPTER 4

TRACTOR PERFORMANCE ON SOFT SOIL - THEORETICAL

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Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.

CHAPTER 4

TRACTOR PERFORMANCE ON SOFT SOIL - THEORETICAL

4.1 INTRODUCTION

4.1.1 General

The study of tractor performance on soft soil is a typical agricultural engineering problem but it is part of a much larger subject that includes soil - implement and soil - vehicle mechanics in general and other applications associated with military and space vehicles[1]. Early work on military vehicles was mainly concerned with the prediction of "trafficability", ie, if a simple penetrometer (a device for measuring the force to push a certain shape into the soil) could be used to predict whether a vehicle could traverse a particular area of ground.

More recent studies of tractor performance on soft soil have proceeded along two lines as mentioned in Section 1.4.2 (c) and (d) above, viz:

(a) Theoretical

The theoretical approach uses classical soil properties (cohesion (c) and angle of internal friction ( )) and some semi - empirical parameters to develop a model for the prediction of the tractive force (soil reaction) and drawbar pull. This approach, which provides the best understanding of the traction process and an appropriate introduction for students, will be followed here.

(b) Empirical

The empirical approach is one where the tractor performance is predicted purely on the basis of a correlation of cone penetrometer readings with corresponding performance measurements. Such an approach provides a ready and useful means of performance prediction but it is not suitable as a basis for understanding the traction process; a brief treatment is given in Chapter 5.

The usual approach to considering the prediction of tractor performance is to begin with the study of the performance of single wheels. The performance of the tractor is then understood as the combined interaction and performance of two or more such wheels.

4.1.2 Definitions

The factors which are significant in the study of the performance of a single wheel may be defined as follows:

(i)   Vertical load or weight on the wheel, W is the vertical force through the axle.

(ii)   Travel (output) speed, V is the linear speed of driven wheel; there is usually some loss in motion due to wheel-slip; thus from Equation 2.1:

$$\text{Travel speed} < \text{Rotational speed} \times \text{Rolling radius}$$

(iii)   Rolling radius is defined in terms of the 'distance traveled per revolution' /2   under some defined zero conditions; these usually include zero drawbar pull, zero braking torque and a defined surface.

(iv)   Wheel-slip, i is the proportional measure by which the actual travel speed of the wheel falls short of (or exceeds) the "theoretical" speed (Equation 2.5).

(v)   Input torque is the (rotational) input effort on driven wheel which is converted to (linear) output effort (force or drawbar pull); there is usually some loss in effort due to the rolling resistance hence from Equation 2.2:

$$\text{Drawbar pull} < \frac{\text{Input torque}}{\text{Rolling radius}}$$

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[1] Other terms used to describe the general field include `off-road locomotion´ and `terra-mechanics´ (earth mechanics).
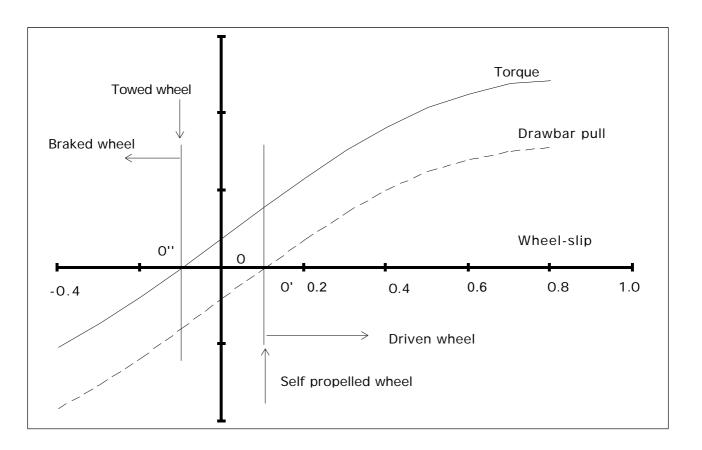
Figure 4.1: Operational states for a wheel; reproduced from Wismer and Luth (1974)
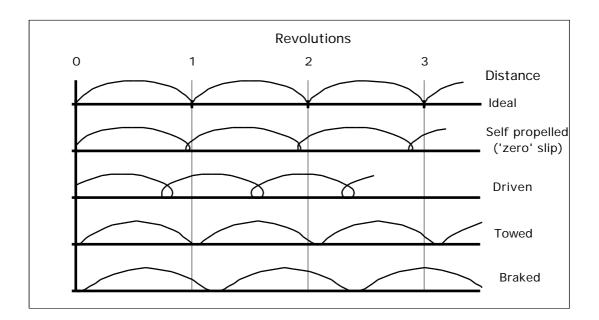with permission of American Society of Agricultural Engineers

Figure 4.2: Motion of a point on a rolling wheel illustrating various conditions of slip; not to scale

(vi)   Rolling (motion) resistance, R is the force opposing motion of the wheel that arises from the non-recoverable energy expended in deforming the surface and wheel. It is convenient to consider this force as acting in the horizontal direction.

(vii)   Tractive force, H is the horizontal reaction <u>on a driven wheel by the soil</u> in the contact area; it is equal and opposite to the horizontal force generated <u>by the wheel on the soil</u>.

(viii)   Drawbar pull, P is the horizontal force at the axle generated by a driven wheel; from Equation 2.7 it may be assumed that:

Drawbar pull = Tractive force - Rolling resistance

(ix)   Towing force is the force to move a freely rolling wheel over the surface and is equal and opposite to the rolling resistance.

The traditional four-wheel tractor is a combination of driven (or occasionally braked) wheels at the rear and free-rolling, towed (pushed) wheels at the front.

4.1.3 Operational states of a wheel

The operation of a wheel can be classified into one of the following states; each occurs within the tractor or other machines under some conditions and each has a particular unknown parameter associated with it.

(a) <u>Towed</u>

Here the wheel, such as the front wheel of the tractor or the wheel of an agricultural implement, is towed with zero opposing external torque; the unknown parameter is the rolling resistance.

(b) <u>Self-propelled</u>

Here the wheel is driven with an external input torque to overcome its own rolling resistance and to propel it across the surface without developing a drawbar pull. This approximates to the drive wheel of a tractor with no drawbar pull (if we neglect the rolling resistance of the front wheels); the unknown parameter is the rolling resistance.

(c) <u>Driven</u>

Here the wheel is driven with an external input torque and is required to develop a drawbar pull as in the drive wheel of a tractor; the unknown parameter is the wheelslip. The extreme case is where the wheel slips, but does not move forward.

(d) <u>Braked</u>

Here the wheel is towed against an opposing, external torque as when being braked or when it is used to generate a torque to operate a 'ground-driven' machine such as a seed drill; the unknown parameter is the wheelslip. The extreme case is where the wheel does not rotate, but just skids across the surface.

Figure 4.1 (Wismer and Luth, 1974) shows these operational states of a wheel in which input and output torque and input and output force (towing force or drawbar pull) are shown plotted against wheel-slip. From this it will be seen that:

(i)   the self-propelled wheel is a special case of the driven wheel, with zero drawbar pull.
(ii)   the towed wheel is a special case of the braked wheel, with zero braking torque.

The origins for the graphs shown are based on the assumption that, with respect to the kinematic ideal (origin at O), a self-propelled wheel is subject to some positive slip (origin at O') and a towed wheel is subject to some negative wheelslip (origin at O").

Figure 4.2 uses the trajectory of a point on a wheel rolling on a horizontal surface (the cycloid) to illustrate the effect of wheelslip by showing the distances traveled by the wheel for the various states discussed above and represented in Figure 4.1. The wheelslip is shown by the loop (motion of the wheel relative to the surface) in the trajectory for the self propelled and driven wheels.

4.1.4 Wheel - slip definition

The generation of a drawbar pull by a wheel driven on a surface results in some relative motion at the wheel - surface interface. This reduces the forward motion of the wheel to less than the ideal value and is referred to generally as 'wheelslip' or 'slip'. In terms of measurement, prediction and presentation of tractor performance, slip is the single most important, dependent parameter.

Slip is defined as the proportional measure by which the actual travel speed (or distance) of a wheel falls short or exceeds the 'ideal' or 'zero' slip speed (or distance). The magnitude of slip is thus dependent on how the `zero´ slip is defined and measured.

The correct zero condition would be under conditions where the travel speed = linear speed of the surface of the wheel = rotational speed x rolling radius. However, since the rolling radius is impossible, or at least difficult, to measure, a more convenient zero condition and method of measuring it is used.   This alternative 'zero' condition is defined as that occurring when the wheel is driven (usually over the (test) surface) with zero drawbar pull (no load), ie, in the self-propelled condition shown as the origin at point O' in Figure 4.1.

Thus as in Section 2.3.1 and Equation 2.5,

$$\text{Wheel-slip, } i = \frac{V_O - V}{V_O} \; 100 \; \% \tag{4.1}$$

where $\quad V_O \quad$ = travel speed when the wheel is driven, with <u>zero</u> drawbar pull, on the surface
$\quad\quad\quad\quad V \quad$ = travel speed when the wheel is <u>generating</u> a drawbar pull, on the surface

The <u>driven</u> condition is used, in preference to the <u>towed</u> (point 0" in Figure 4.1), because it is usually more convenient to drive the wheel over the surface with zero drawbar pull than to tow it.

An alternative 'zero' condition for slip is where, for the zero pull test, the wheel is driven on a <u>hard</u> surface (such as a road), rather than on the <u>test</u> surface. Under these conditions the slip at zero drawbar pull on the test surface will not be zero. In describing an experiment it is necessary to state which 'zero' slip condition was used.

In measuring the performance of a tractor it is not possible to drive a wheel alone over the test surface, hence the zero slip condition is usually taken when the <u>tractor</u> is driven with zero drawbar pull over the test surface. The drive wheels will suffer some extra small slip in having to overcome the rolling resistance of the front wheels. Thus the `zero´ point will be even further to the right than 0' in Figure 4.1.

4.1.5 Wheelslip measurement

The use of velocity for measuring slip for a tractor as described above is not particularly convenient because variations in engine speed would influence the result, hence other methods have been devised. In the following it is assumed that the zero drawbar pull distance is measured on the test surface.

(a) <u>Measurement of distance traveled</u>

In terms of distances (for a given number of wheel revolutions):

$$\text{Wheelslip, } i = \frac{m_O - m}{m_O} \; 100 \; \% \tag{4.2}$$

where: $\quad m_O \quad$ = distance traveled when the tractor is driven with <u>zero</u> drawbar pull on the surface
$\quad\quad\quad\quad m \quad$ = distance traveled when the tractor is <u>generating</u> a drawbar pull on the surface

This is a convenient method when only a distance measuring tape is available and when the counting of whole numbers of wheel revolutions can be done visually; the tractor is tested over the same <u>number of revolutions</u> for both tests.

(b) <u>Counting of wheel revolutions</u>

In terms of numbers of wheel revolutions (for a given distance traveled):

$$\text{Wheel-slip, } i = \frac{N - N_0}{N} \; 100 \text{ \%} \tag{4.3}$$

where: N_0 = number of wheel revolutions when the tractor is driven with <u>zero</u> drawbar pull on the surface

            N = number of wheel revolutions when the tractor is <u>generating</u> a drawbar pull on the surface

This is a convenient method if equipment to measure fractions of a wheel revolution is available; the tractor is tested over the same <u>distance</u> in both tests.

(c)   <u>Use of a free rolling wheel</u>

On some occasions it is desirable to be able to measure slip while moving but to do so it is necessary to avoid the requirement that the zero-pull test and subsequent with-pull tests be conducted over the same number of wheel revolutions (method (a) above) or for the same distance (method (b) above).

The use of a free-rolling wheel (such as an attached 'fifth wheel' or a tractor front wheel) as a `non-slip´ reference overcomes this problem in principle. The method involves the use of revolutions of the free wheel (n_0 and n) to infer the rear wheel revolutions under the zero-pull (N_0) test, corresponding to the unknown distance used for the pull test (for which N revolutions were recorded). Thus from Equation 4.3,

$$\text{Wheelslip, } i = \frac{N - \dfrac{n}{n_0} N_0}{N} \tag{4.4}$$

From Equation 4.4, it can be seen that the rear wheel revolutions (N_0) for the zero pull tests are scaled by the ratio of the free wheel revolutions for the zero-pull and with-pull tests, n_0 and n, to give the zero pull, rear wheel revolutions corresponding to the pull test distance.

In order to use this method it is necessary to have a wheel counter (to measure fractions of a revolution) on both the driving and the free wheel(s). It should also be noted that the free wheel revolutions are affected by speed and surface condition and so the free wheel may need to be calibrated if accurate results are to be obtained, particularly at small slips (Parkhill and Macmillan, 1984).

Modern techniques for continuously measuring slip using radar or ultrasonic sound for speed measurement are now available.

4.2 TRACTIVE PERFORMANCE

As discussed in Section 1.4 above, four different approaches have been taken to the study of tractor performance; three have been applied to tractive performance.

4.2.1 Practical / experimental measurement

The early study of the performance of tractors was limited to the experimental measurement of travel speed and wheelslip at various drawbar pulls on soils (for example, Southwell, 1964) and on test tracks (Baillie and Vasey, (1969) . The results, as discussed in Chapter 3, were intended to provide an understanding of the principles involved and a basis for comparing the relative performance that farmers might expect from the various tractors in the field.

Rolling resistance of wheels was measured by equating it to the towing force required to move different types of (mainly transport) wheels across visually described surfaces, eg. road (hard), stubble (firm), cultivated soil (soft) etc. The results were quoted on the basis of a coefficient of rolling resistance. The early work of McKibben and Davidson, (1940) was of this type.

4.2.2 Theoretical prediction

The theoretical prediction of tractive performance has involved the separation of the problem into two parts, viz, the prediction of:

  (i) tractive force, H
  (ii) rolling resistance, R

Using this approach it is assumed (Equation 2.7) that the drawbar pull (P) is what remains of the tractive force after the rolling resistance has been overcome, ie:

$$P(i) \;=\; H(i) \;-\; R \tag{4.5}$$

where: P(i) implies that P will be determined as a function of slip
       H(i) implies that H will be predicted as a function of slip

While R is also a function of slip, this function is not known and hence the value for R is that measured under the towed condition or predicted using the theory in Section 4.3, both of which assume zero slip. Clearly this is only approximate because the rolling resistance under finite slips will be greater than the value measured or predicted with zero slip.

The generation of a tractive force by the tractor requires an equal and opposite horizontal reaction by the soil against the driving wheels in the contact area. This reaction force, which in effect determines the tractor performance, is predicted on the basis of the soil strength parameters (c and) and the soil deformation corresponding to various wheelslip values.

The support of the tractor requires a vertical reaction on the wheels which causes vertical deformation of the soil in the contact area. Equating the energy to deform the soil (ie. to make the rut) to the work done by the rolling resistance force provides a basis for calculation of the latter. The process is modelled by the pressure - sinkage relationship for a plate pressed into the soil; slip is considered to be zero (See Section 4.3).

4.2.3 Empirical prediction

Here experimental data on the drawbar pull and rolling resistance of various wheels together with a single soil parameter (the cone index obtained by measuring the force to push a cone penetrometer into the soil) are used to predict drawbar pull and rolling resistance on a purely empirical basis (Wismer and Luth, 1974) as discussed in Chapter 5.

As mentioned in Section 1.4 above the theoretical / predictive approach provides the best basis for understanding tractive performance and will be emphasised here; the other approaches may be more readily used for the immediate determination of wheel performance.
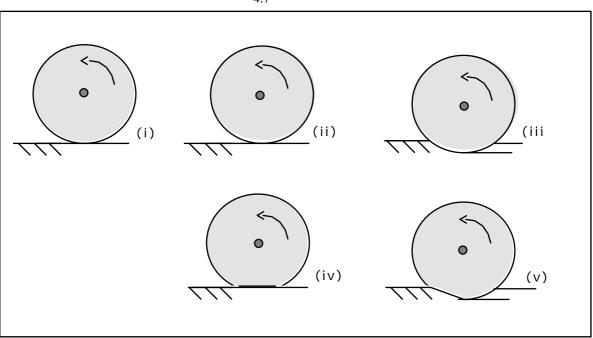
Figure 4.3: Various conditions for a wheel rolling on a surface

4.3 ROLLING RESISTANCE

4.3.1 Wheel conditions

The rolling resistance of a wheel is, in general terms, the force opposing the motion of the wheel as it rolls on a surface. This force arises from the energy losses that occur due to

(i)   the elastic but non-ideal deformation of the wheel
(ii)   the inelastic and non-recoverable (plastic) deformation of the surface
(iii)   friction in the wheel bearings (usually assumed to be negligible)

From this it will be clear that the rolling resistance of a wheel will be a function of the strength - deformation properties of the surface and the size and deformation characteristics of the wheel. For wheels with tyres, the secondary factors include the air pressure, the structure of the tyre carcass (radial or bias ply) and the tread pattern.

For speeds used with agricultural tractors, rolling resistance is relatively independent of the speed of deformation of the soil and the tyre, hence of the travel speed.

We may consider a range of wheels as shown in Figure 4.3; here 'hard' means near rigid and 'soft' means deformable.

(i)   The ideal is a perfectly rigid wheel rolling on a perfectly rigid surface. This defines the kinematics of the rolling wheel.

(ii)   Hard wheel on a hard surface. This is approximated to by an elastic steel wheel rolling on an elastic steel track as in a railway.

(iii)   Hard wheel on soft surface. Here most of the deformation and energy loss occurs in the surface which yields plastically but does not recover. Tractor front wheels and implement wheels with 'high' pressure tyres, operating on soft agricultural soil, are of this type.

(iv)   Soft wheel on hard surface. Here most of the deformation and energy loss occurs in the wheel and appears as heat. Tractor driving wheels and vehicle wheels both operating on road surfaces are of this type.

(v)   Soft wheel on soft surface. Here both the wheel and the surface deform significantly as in the tractor rear wheel operating on soft soil. Energy loss occurs mainly in deforming the soil as in (iii) above.

One major aspect of understanding and predicting tractor performance is that of determining the rolling resistance of a wheel as it is towed without slip over the surface. The problem of determining the rolling resistance of a driving wheel, when slip is present, is more complex and will not be considered here (Reece, 1965-66).

4.3.2 Theoretical prediction

When a wheel rolls over a soft surface it makes a rut or compacted track. The simplest basis for the prediction of its rolling resistance is to therefore assume that the work done against the rolling resistance is the work done in compacting the soil. Bekker (1956) assumed that the wheel was equivalent to a plate continuously being pressed into the soil to a depth equal to the depth of the rut produced by the wheel.

(a) Work done to deform soil

For a plate, length ℓ, width b, being pressed into the soil, as in Figure 4.4, Bekker suggested that the pressure, p under such a plate is given by:

$$p = (\frac{k_c}{b} + k) z^n \qquad (4.6)$$

where: z is vertical soil deformation (sinkage)
             k_c , k   are soil sinkage moduli
             n is soil sinkage exponent
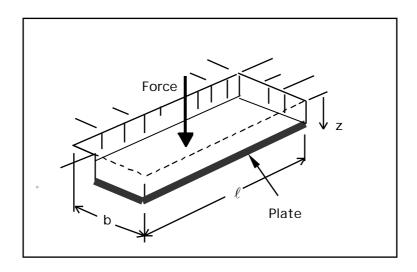   b is the width of the plate

Figure 4.4: Plate being pushed into the soil to measure rolling resistance parameters (Cut away view)
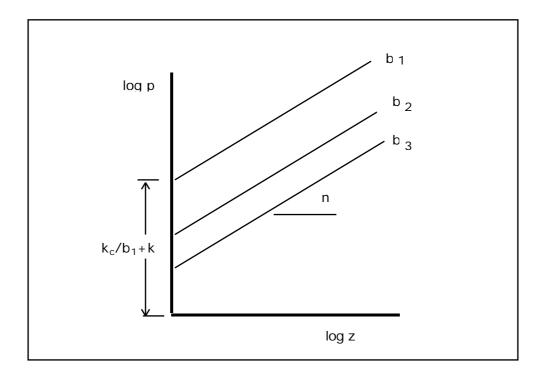
Figure 4.5: Log p plotted against log z in analysis of plate sinkage tests.
Reproduced from Bekker (1969) with permission of University of Michigan Press

Then the vertical work to press such a plate into the soil:

$$\text{Work} = b\ell \int_{0}^{z_0} p \; dz$$

$$= b\ell \; (\frac{k_c}{b} + k \;) \int_{0}^{z_0} z \; dz$$

$$= \frac{\ell \; (k_c + bk \;)}{n+1} \; z_0^{\,n+1} \tag{4.7}$$

But for a weight, W on the plate, at maximum sinkage z_0,

$$W = b\ell \; p_{max}$$

$$= b\ell \; (\frac{k_c}{b} + k \;) \; z_0^{\,n}$$

$$= \ell \; (k_c + bk \;) \; z_0^{\,n}$$

$$z_0 = \left[\frac{W}{\ell \; (k_c + bk \;)}\right]^{1/n}$$

Substituting for z_0 in Equation 4.7 gives

$$\text{Work} = \frac{\ell \; (k_c + bk \;)}{n+1} \; \left[\frac{W}{\ell \; (k_c + bk \;)}\right]^{(n+1)/n} \tag{4.8}$$

Before considering the two types of wheel / surface that have been analysed on this basis we need to show how the soil parameters can be measured.

(b) Measuring soil parameters

Because the work to compact the soil is used as the basis of prediction of rolling resistance, the force to push a plate into the soil and the associated sinkage is chosen as an appropriate method of determining the soil parameters for the calculation of rolling resistance.

To obtain the parameters, a series of plates of different widths, b_1, b_2, b_3 are pushed into the soil while the force and corresponding sinkage are measured. From Equation 4.6 we can write:

$$\log p = \log (\frac{k_c}{b} + k \;) + n \cdot \log z$$

Assuming the data follow Equation 4.6, when log p is then plotted against log z, we get a series of straight lines of slope 'n' and intercept on the log p axis $= (\frac{k_c}{b} + k \;)$ as shown in Figure 4.5. Further if the intercepts are then plotted against $\frac{1}{b}$ the slope of this line is k_c and the intercept at $\frac{1}{b}$ is k .
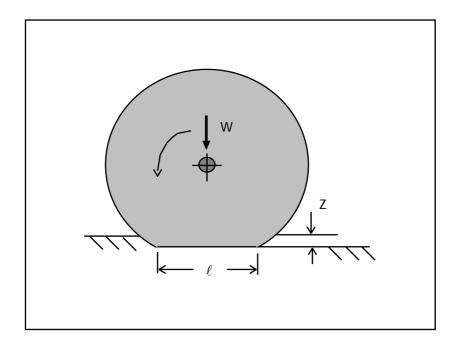
Figure 4.6: Parameters for the analysis of the rolling resistance of a soft wheel on a soft surface.
Reproduced from Bekker (1960) with permission of the University of Michigan Press.
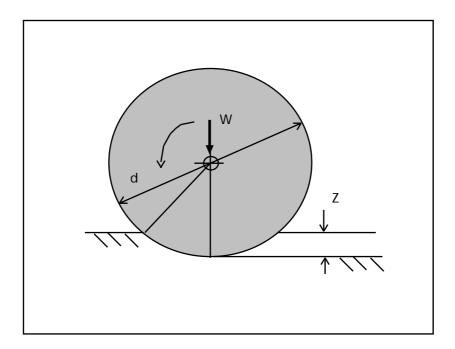

Figure 4.7: Parameters for the analysis of the rolling resistance of a hard wheel on a soft surface.
Reproduced from Bekker (1956) with permission of University of Michigan Press.

(c) <u>Soft wheel on soft surface</u>

Here the wheel (or a track) is assumed to impose a uniform pressure on the soil which deforms uniformly over the contact area (as in Figure 4.6(a)) until the contact area times the pressure at the tyre surface is equal to the weight on the tyre. This pressure may be assumed to be made up of the pressure equivalent to the stiffness of the tyre carcass and the internal pressure of the air (and the water if used).

Consider the work done in towing such a wheel a distance, ℓ against the rolling resistance, R. In simple terms, if this is equal to the work done on forming the rut as calculated for the plate, length ℓ, width b pressed into the soil, as in (a) above:

$$R\,\ell \;=\; \frac{\ell\,(k_c + bk)}{n+1}\left[\frac{W}{\ell\,(k_c + bk)}\right]^{(n+1)/n}$$

Thus the rolling resistance,

$$R \;=\; \frac{(k_c + bk)}{n+1}\left[\frac{W}{\ell\,(k_c + bk)}\right]^{(n+1)/n}$$

$$= \frac{1}{(n+1)(k_c b + bk)^{1/n}}\left[\frac{W}{\ell}\right]^{(n+1)/n} \qquad (4.9)$$

Writing this in terms of the ground pressure $p = \dfrac{W}{b\ell}$ gives:

$$R \;=\; \frac{b}{(n+1)\left(\dfrac{k_c}{b} + k\right)^{1/n}}\,(p)^{(n+1)/n} \qquad (4.10)$$

This simple analysis suggests that rolling resistance depends directly (but not necessarily proportionally) on the weight on the wheel W, and inversely (but not necessarily proportionally) on the length of the contact area, ℓ but not the diameter of the wheel except in so far as it affects ℓ. It also depends in a complex way on the width of the contact area, b.

For n = 1, which might be considered typical for an agricultural soil (Dwyer, 1984), this equation can be put in the form of a coefficient of rolling resistance (see Section 4.3.3):

$$\frac{R}{W} \;=\; \;=\; \frac{p}{2\ell\left(\dfrac{k_c}{b} + k\right)} \qquad (4.11)$$

This equation suggests that the coefficient of rolling resistance will be proportional to the ground pressure and inversely proportional to the length of the contact area. Hence, for example, improved traction will be achieved on sandy soils if p is small and ℓ is large, ie, by the use of low pressure tyres.

d) <u>Rigid wheel on a soft surface</u>

Here the problem, as shown in Figure 4.6(b), is more complex because the sinkage and hence the pressure is not constant over the contact area as was assumed for the uniform sinkage case above. It can be shown (Bekker 1956) that:

$$R = \frac{\left[\dfrac{3W}{(3-n)\sqrt{D}}\right]^{\frac{2n+2}{2n+1}}}{(n+1)(k_c + bk)^{\frac{1}{2n+1}}} \qquad (4.12)$$

Here it will be seen that the rolling resistance is dependent, in a complex way, on the weight on the wheel as well as its width and <u>diameter</u> compared with the <u>length of the contact patch</u> in the previous analysis.
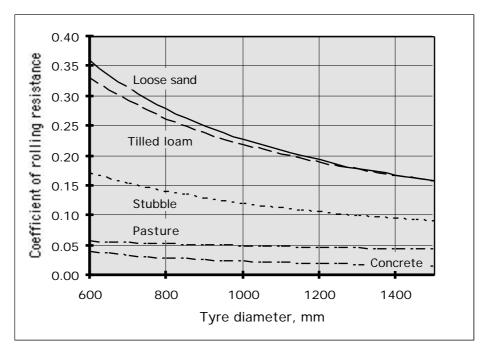
Figure 4.8: Rolling resistance of agricultural tyres of different diameter on various surfaces.
Reproduced from McKibben and Davidson (1940) with permission of the
American Society of Agricultural Engineers

4.3.3 Experimental measurement

Historically the experimental measurement of rolling resistance provided the data for the evaluation of traction systems. The weight on the wheel, the wheel diameter and / or width and the soil condition were seen as the most important factors and so the rolling resistance for each type of wheel was expressed in terms of the dimensionless number:

$$\text{Coefficient of rolling resistance,} \quad = \frac{\text{Rolling resistance force}}{\text{Weight force}} \qquad (4.13)$$

Use of such a coefficient requires that the wheels must be defined in terms of their diameter, width, etc, and soil conditions be verbally described.

The early work of McKibben and Davidson (1940), as shown (corrected) in Figure 4.8, used this approach. The intuitive and practical experience that we have of the significance of wheels rolling on soft surfaces is confirmed by that graph. There it will be seen that the coefficient for sand and loose soil is some 4 - 6 times that for concrete and firm soil and that doubling the diameter will halve the coefficient.

4.3.4 Empirical prediction

 The empirical prediction of rolling resistance is considered in Chapter 5

4.4 TRACTIVE FORCE

4.4.1 Introduction

A track or wheel generates a tractive force by reacting (pushing) against the soil. Any such force involves shear stresses in, and an associated deformation between the track (together with the soil between its lugs or grousers) and the underlying soil bulk. For the track as a whole such deformation results in slip or lost motion. An analysis of the generation of tractive force therefore requires a knowledge of the stress - deformation relationship of the soil.

4.4.2 Shear stress - deformation characteristic for soil

The shear stress - deformation relationship for soils may take different forms depending on the normal and shear stresses under which they were compacted and their degree of cementation (bonding together of the soil particles). Bekker (1956) fitted empirical equations to two typical forms and analysed tractive force by integrating them over the length of the track. Only the simpler analysis applicable to loose and / or non-cemented soil with slowly rising shear stress - deformation characteristic (as shown in Figure 4.9) will be given here.

The soil shear stress - deformation characteristic for such a soil is assumed to have the following form:

$$S \quad = \quad S_{max} \, (1 - e^{-j/k} \,) \qquad\qquad (4.14)$$

where S_{max} = shear strength of the soil and corresponds to shear stress at large deformation

$$= (c + \quad \tan \quad)$$

c	= soil cohesion
	= angle of internal friction
	= normal stress
j	= shear deformation
k	= shear deformation modulus

Hence $S \quad = (c + \quad \tan \quad) (1 - e^{-j/k}) \qquad\qquad (4.15)$
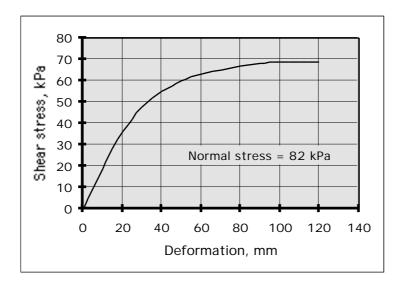
Figure 4.9: Typical shear stress / deformation curve for a loose uncemented soil.
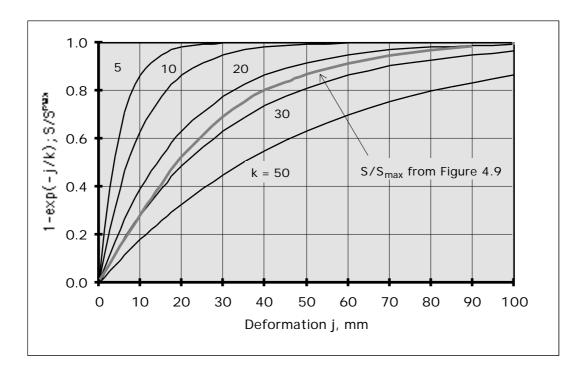

Figure 4.10: Plot of $e^{-j/k}$ and S/S_{max} from Figure 4.9

The shear deformation modulus indicates the 'rigidity' or deformation at which the soil reaches its shear strength in being sheared. It is a characteristic dimension and is taken as that at which the shear stress reaches 95% of its final value (Wills, 1963) as shown in Figure 4.10.

ie.
$$\frac{S}{S_{max}} = 0.95 = (1 - e^{-j/k})$$

$$e^{j/k} = 20 \quad \text{ie,} \quad \frac{j}{k} = \ln 20$$

$$k = \frac{j}{3} \quad \text{(closely)}$$

Thus k is 1/3 of the deformation corresponding to 95% of the maximum shear stress.

To determine k, it is necessary to measure the shear stress (S) - deformation (j) characteristic for the soil and then to plot the following against j:

(i) $\dfrac{S}{S_{max}}$ from experimentally measured results (Figure 4.9, Pudjiono (1998))

(ii) $1 - e^{-j/k}$ for different assumed values of k, as shown plotted in Figure 4.10.

The modulus k may then be chosen by inspection according to the value corresponding to that graph (ii) which best fits (i). Other methods are discussed by Wills (1963).

4.4.3 Analysis of locked track

Consider a rigid, inextensible track as shown in Figure 4.11 standing on a soil with strength parameters, cohesion (c) and angle of internal friction ( ) and with a rising stress - deformation characteristic, as given in Figure 4.9. Assume track grousers of width b, length ℓ and carrying a weight W, are engaged in the soil.

If the track is locked, the maximum tractive force that the track can generate will be the maximum force the soil can resist.

$$H_{max} = \text{Area } S_{max}$$

$$= b\,\ell\,(c + \tan\ )$$

$$= b\,\ell\,c + b\,\ell\ \tan$$

$$H_{max} = Ac + W \tan \tag{4.16}$$

This neglects any contribution of the soil being sheared at the end of the grousers.

H_{max} represents the absolute maximum capacity of the track at large soil deformation corresponding (approximately) to 100% slip. According to this simple theory, it is an upper-bound value that may be approached but never exceeded.

This equation implies that H_{max} depends on:

(i) the area of the track which contributes to H_{max} through the cohesive strength of the soil
(ii) the weight on the track which contributes to H_{max} through the frictional strength of the soil

Dividing by W gives, in a similar way to Equation 2.15,

$$' = \frac{H_{max}}{W} = \frac{c}{\dfrac{W}{A}} + \tan$$

$$= \frac{c}{\ } + \tan \tag{4.17}$$

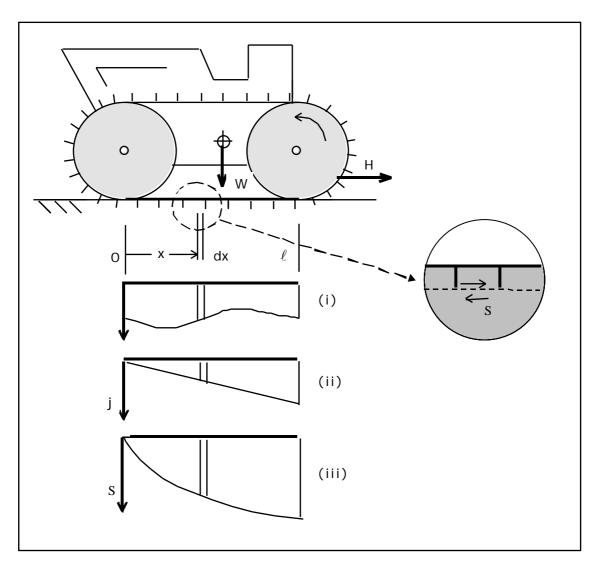where ' is a 'gross' tractive coefficient.

Figure 4.11 Operational parameters for a track showing the variation along the track of:
(i) normal stress,   ; (ii) horizontal deformation, j; shear stress, S.

Problem 4.1

Figure 4.12 shows a crawler tractor standing (a) on level ground and (b) on a slope.

The following data apply:

Track - soil contact length	ℓ	= 1.2 m
Track width (total for two)	b	= 0.6 m
Tractor mass	W	= 2.4 T
Soil cohesion	c	= 15 kPa
Soil angle of internal friction		= 30^0
Angle of slope		= 15^0

Estimate the capacity, H, of the tractor as an anchor and the gross tractive coefficient, μ; assume that the normal stress under the track is uniform.

Solution (b):

Resolving along the slope:

$$H \cos\alpha + W \sin\alpha = Ac + (W \cos\alpha - H \sin\alpha) \tan\phi$$

$$H \cos\alpha + H \sin\alpha \tan\phi = Ac + W \cos\alpha \tan\phi - W \sin\alpha$$

$$H = \frac{Ac + W(\cos\alpha \tan\phi - \sin\alpha)}{(\cos\alpha + \sin\alpha \tan\phi)} = \frac{1.2 \times 0.6 \times 15 + 23.5 (\cos15 \tan30 - \sin15)}{(\cos15 + \sin15 \tan30)}$$

$$= 16 \text{ kN}$$

$$\mu = \frac{H}{W\cos\alpha - H\sin\alpha} = \frac{16}{18.6} = 0.86$$

Answers: (a) 24.4, 1.04

Repeat for other arrangements where H is neither along the slope nor horizontal.

Problem 4.2

A rubber wheel carrying a load W of 5.4 kN has an effective ground contact area A of 0.09 m^2 over which the pressure may be assumed to be uniform. The soil and rubber / soil strength characteristics are shown in Figure 4.13

What is the maximum pull which can be generated by the wheel if:
      (i) the wheel has lugs which engage the soil?
      (ii) the lugs are removed?

Solution (i):

$$\sigma = \frac{5.4}{0.09} = 60 \text{ kPa}$$

$$H_{max} = \text{tractive force at the contact area}$$
$$= 36 \times 0.09$$
$$= 3.24 \text{ kN}$$

Alternatively the strength of the soil may be calculated as $Ac + W \tan\phi$.

$$H_{max} = 0.09 \times 20 + 5.4 \times 0.267 = 1.8 + 1.44 = 3.24 \text{ kN}$$
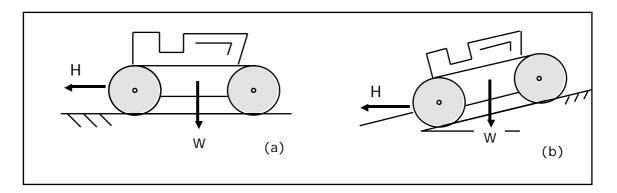
Answer (ii): 1.57 kN

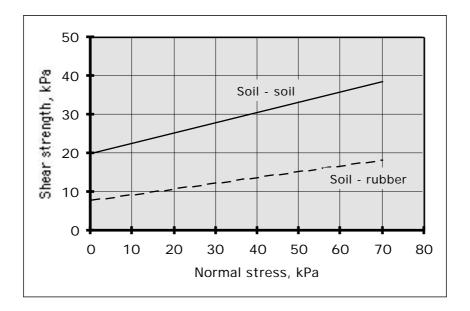Figure 4.12: Tractor as an anchor in Problem 4.1

Figure 4.13: Soil and rubber characteristics for Problem 4.2

4.4.4 Analysis of track with slip (Bekker, 1956)

Consider the track in Figure 4.11 being driven over a soil surface while developing a drawbar pull = tractive force H. The rotation of the track is such that a length of track equal to the track wheel centre distance (ℓ) is laid out; this is equivalent to a fraction of a revolution. Before the track moves an element of soil at its front will have zero deformation; after the track has passed over it will have a finite value, j_{max}.

From Equation 4.2 above,

$$\text{Track slip, } i = \frac{m_o - m}{m_o}$$

For no tractive force, the movement of the track forward will be equal to the wheel centre distance, ie. $m_o = \ell$

With tractive force the movement of the tractor = m

$$i = \frac{\ell - m}{\ell}$$

But (ℓ - m) = maximum distance moved rearwards by the soil, ie, j_{max}.

$$i = \frac{j_{max}}{\ell}$$

But since the track is inextensible, the deformation must grow linearly from front to rear as shown in Figure 4.11.

$$i = \frac{j}{x}$$

$$j = i\,x \qquad\qquad (4.18)$$

Tractive force is the sum of the contributions of the shear stress (times the corresponding area) for all the elements of soil along the track :

$$H = b \int_0^{\ell} S\, dx$$

$$= b \int_0^{\ell} (c + (x)\tan))\ (1 - e^{-j/k})\ dx$$

where (x) represents as a function of x.

If it is assumed that is constant, ie, independent of x,

$$H = b(c + \tan) \int_0^{\ell} (1 - e^{-j/k})\ dx$$

$$= b(c + \tan) \int_0^{l} (1 - e^{-ix/k})\ dx$$

$$= b(c + \tan) \left[x + \frac{k}{i}\, e^{-ix/k} \right]_0^{\ell}$$

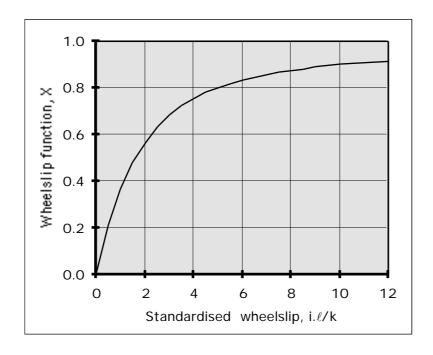$$= b(c + \tan) \left[\ell + \frac{k}{i}\, e^{-i\ell/k} + \frac{k}{i} \right]$$

Figure 4.14: Slip function, X versus iℓ/k; reproduced from Reece (1967) with permission of the Institution of Agricultural Engineers.

$$4.22$$

$$= \quad b\,\ell\,(c + \quad \tan \quad)\ \left[\ 1 - \frac{k}{i\ell} \ + \ \frac{k}{i\ell}\ e^{-i\ell/k}\ \right]$$

$$= \quad (Ac + W \tan \quad)\ \left[\ 1 - \frac{k}{i\ell} \ + \ \frac{k}{i\ell}\ e^{-i\ell/k}\ \right]$$

$$= \quad H_{max} \ . \ X \qquad\qquad\qquad\qquad (4.19)$$

where X   = slip function for assumed constant normal stress.

$$= \quad \left[1 - \frac{k}{i\ell} \ + \ \frac{k}{i\ell}\ e^{-i\ell/k}\right] \qquad\qquad\qquad (4.20)$$

The slip function X is shown plotted against $\dfrac{i\ell}{k}$ in Figure 4.14. This is a slip - tractive force graph where $\dfrac{i\ell}{k}$ is a 'standardized slip' and X is the corresponding function giving H in terms of H_{max}.

The following terms are significant in the contributions that they make to the tractive force.

   (i)   c and tan : the soil strength parameters contribute to H through their contribution to H_{max}.

   (ii)   A (= b . ℓ) : the track area contributes to H through the contribution of the cohesive component of the soil strength to H_{max}; it will be proportional to A for a purely cohesive soil for which   = 0.

   (iii)   W (= b . ℓ .   ): the weight contributes to H through the contribution of the frictional component of the soil strength to H_{max}. ; it will be proportional to W for a frictional soil for which c = 0.

   (iv) ℓ : the track length contributes to H through its contribution to track area as explained above. It also contributes as it appears in the slip function in a way that causes an increase in X as length increases; thus track length has significant effect on H in addition to its area effect.

   (v)   k: decreasing the horizontal deformation modulus (having a more rigid soil that reaches its maximum shear stress at lesser deformations) has the effect of increasing H by causing an increase in X.

   (vi)   i: increasing the slip increases the deformation and the associated shear stress, which has the effect of increasing X and H.

The above analysis may be extended to a wheel if it is assumed that the pressure under the wheel is constant. The area of the contact patch may be assumed to be 0.78 b ℓ.

4.5 DRAWBAR PULL

The above gives the tractive force - slip relationship for a track or wheel. It is clear that it also gives the basic form to the drawbar pull - slip relationship for the performance of tractors measured in the field where the drawbar pull is what remains of the tractive force after the rolling resistance has been overcome.

Fig. 4.15 shows the comparative performance of the same basic tractor (New Fordson Major) with different wheel equipment, viz, two wheel drive (2WD) , four wheel drive (4WD) and tracks on cultivated (loose) and stubble (firm, rigid) soil (Anon., undated).

From these results it is clear that the following give reduced slip and increased drawbar pull:

(i)   tracks compared to 2WD on cultivated (loose) soil which shows the effect of area and length of contact patch and of weight

(ii)   tracks on stubble (firm) compared to cultivated (loose) soil which shows the effect of soil strength and rigidity (deformation modulus)

(iii)   4WD compared to 2WD on cultivated (loose) soil which shows the effect of area and weight.
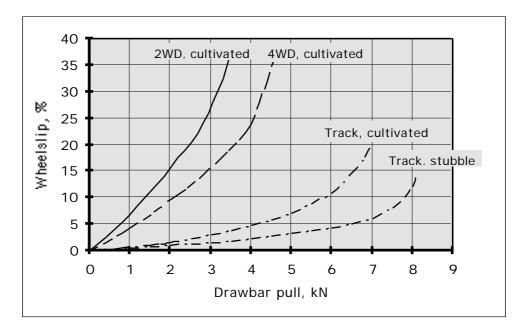
Figure 4.15 Comparative slip - pull performance of two wheel drive, four wheel drive and tracked tractor. Reproduced from Anon. (undated) with permission of Silsoe Research Institute.
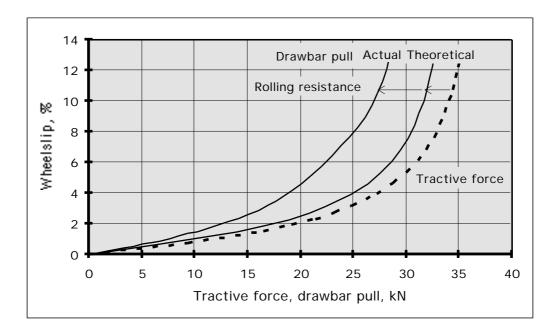

Figure 4.16: Tractive force, actual and predicted drawbar pull for Problem 4.3

As noted above, R is usually measured or predicted on the assumption of zero slip. An increase in slip associated with an increase in H will mean that the prediction becomes less accurate because the above assumption will be increasingly invalid.

Hence the prediction of P using Equation 4.5:

$$P = H - R \qquad\qquad (4.5)$$

will also become less accurate as shown in Problem 4.3, Figure 4.16.

Problem 4.3

Figure 4.16 shows a plot of measured wheel slip - drawbar pull data for a small crawler tractor tested on soil. The following data apply:

Tractor:		**Soil** (assumed):	
Weight, W	= 3800 kg	Soil cohesion, c	= 15 kPa
Track length, ℓ	= 1.65 m	Soil angle of friction,	= 30°
Track width, b	= 0.35 m	Soil deformation modulus, k	= 0.02 m
Rolling resistance, R	= 2.5 kN		

Calculate the theoretical wheel slip - drawbar pull performance and plot it on the graph with the performance from the actual test.

$$H_{max} = Ac + W \tan$$
$$= 2 \times 0.35 \times 1.65 \times 15 + 3.8 \times 9.8 \times \tan 30 = 38.8 \text{ kN}$$

For appropriately chosen values of i, calculate $\dfrac{i\ell}{k}$ hence X from Equation 4.20 or read from

Figure 4.14. Calculate H from Equation 4.19 and P from Equation 4.5 above. The results are shown plotted in Figure 4.16

4.6 DRAWBAR POWER

4.6.1 Wheel-slip - drawbar power characteristic

While the wheel-slip - drawbar pull graph above is the main performance characteristic for a track (or wheel) the user is, however, usually more concerned with work rates, ie, drawbar power. The drawbar power - slip results (of the tractor tests shown in Figure 4.15) have been plotted in Figure 4.17 and show that there is an optimum slip that gives a maximum drawbar power. Since all of these tractors had the same engine power, Figure 4.17 also shows (in relation to maximum drawbar power) how significant the soil condition is (track - stubble compared to track - cultivated) and also wheel / track contact area and weight are (track compared to 2WD and 4WD each for cultivated soil).

Figure 4.17 also shows the much greater power obtained from the track (and to a lesser extent the 4WD) due to the larger drawbar pull that can be achieved without excessive slips and the losses in power that are associated with them.

The wheelslip - drawbar power characteristic may be plotted from experimental data as shown in Problem 4.4.
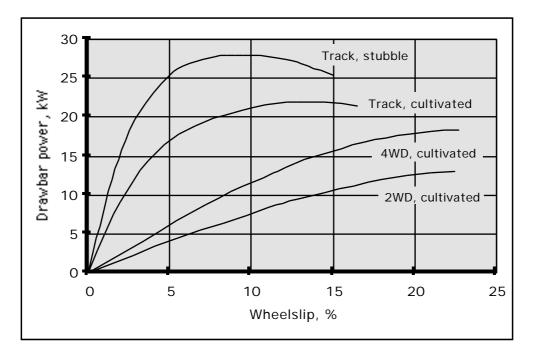
Figure 4.17: Comparative power – slip performance of two-wheel drive,
four-wheel drive and tracked tractor. Reproduced with permission of
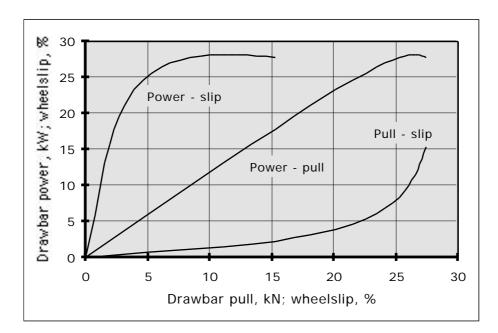Silsoe Research Institute

Figure 4.18 Slip- drawbar power - drawbar pull performance for Problem 4.4

Problem 4.4

Figure 4.18 shows the track slip - drawbar pull graph for a tracked tractor. The following data apply:

Diameter of track   = 0.85 m
Engine speed   = 1760 rpm (assumed constant)
Overall gear ratio   = $\dfrac{1}{65}$

Plot:
    (i) drawbar power versus drawbar pull
    (ii) drawbar power versus track slip, hence
    (iii determine the conditions for maximum power

Linear speed of track $V_O = $ D N $= \dfrac{3.14 \times 0.85 \times 1760}{65 \times 60} = 1.2$ m/s   (Equation 2.1)

Travel speed, V $= 1.2$ (1-i) where the slip, i is read from Figure 4.18 for various drawbar pulls
Drawbar power, Q $= $ P.V as plotted in Figure 4.18

The maximum power of 28 kW is achieved at a drawbar pull = 26 kN and track slip = 11 %

4.6.2 Theoretical prediction of optimum wheel-slip

The performance of a track may be best characterized by the drawbar pull and slip at maximum drawbar power; this may be predicted as follows (Reece 1967).

From Equations 4.5:

$$P = H - R$$

$$= (Ac + W \tan\ )\ X - R \qquad\qquad (4.21)$$

Drawbar power:
$$Q = P.V$$

But from Equation 2.6

$$V = V_O (1-i)$$

$$Q = V_O (1 - i)\ \big[(Ac + W \tan\ )\ X - R\big] \qquad\qquad (4.22)$$

where V_O = tractor wheel speed.

In order to determine the slip for maximum drawbar power by differentiation, it would be necessary to know R as a function of slip. This is not available so an alternative is to neglect the influence of R relative to H and to calculate slip for maximum tractive power Q´, ie, obtain a maximum for:

$$Q' = H.V$$

$$= V_O(1- i)\ (Ac + W\ \tan\ )\ X$$

$$= V_O (1- i)\ (Ac + W \tan\ )\ \Big[1 - \frac{k}{i\ell} + \frac{k}{i\ell}\ e^{-i\ell/k}\Big]$$

$$= V_O (Ac + W \tan\ )\Big\{ \Big[1 - \frac{k}{i\ell} + \frac{k}{i\ell}\ e^{-i\ell/k}\Big] - \Big[i - \frac{k}{\ell} + \frac{k}{\ell}\ e^{-i\ell/k}\Big] \Big\}$$
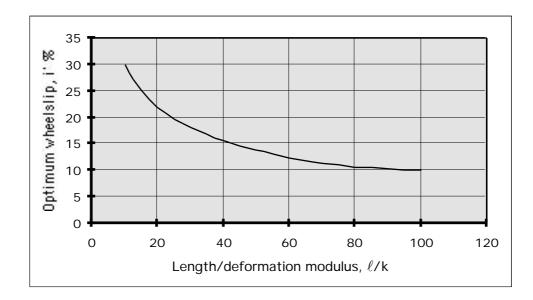
Figure 4.19: Optimum wheelslip as a function of track length/deformation modulus.
Reproduced from Reece (1967) with permission of Institution of Agricultural Engineers.
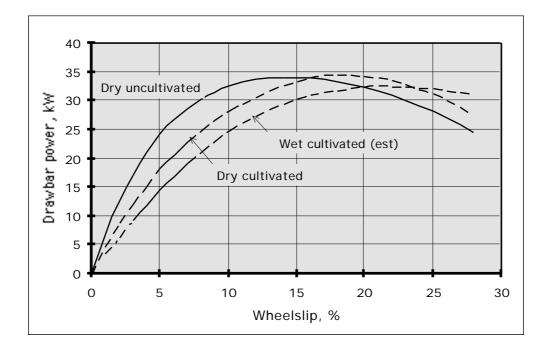

Figure 4.20: Drawbar power - wheelslip performance of a tractor on soil in different conditions.
Reproduced from Hutchings (1980) with permission of Department of Natural Resources and Environment (Vic)

Differentiating this with respect to slip gives:

$$\frac{dQ'}{di} = V_o (Ac + W \tan \phi) \left\{ \left[\frac{k}{i^2\ell} - \frac{1}{i} e^{-i\ell/k} - \frac{k}{i^2\ell} e^{-i\ell/k} \right] - \left[1 - e^{-i\ell/k} \right] \right\}$$

$$= 0$$

ie, $$\frac{k}{i^2\ell} + e^{-i\ell/k} \left[1 - \frac{k}{i^2\ell} - \frac{1}{i} \right] - 1 = 0 \qquad (4.23)$$

Reece gives the numerical solution to this equation in Figure 4.19 and uses it to give, in terms of the ratio ℓ/k, an approximation to the slip i´, at which maximum drawbar power is obtained.

This relationship suggests that the slip i´, at maximum drawbar power (strictly maximum tractive power) decreases:

(i)   as k, the deformation modulus decreases, ie, the soil becomes more rigid and approaches its maximum shear stress at smaller deformations

(ii)   as ℓ, the length of the contact area, increases

The drawbar power - slip results (of the tractor tests shown in Figure 4.15) which have been plotted in Figure 4.19 confirm this prediction, viz:
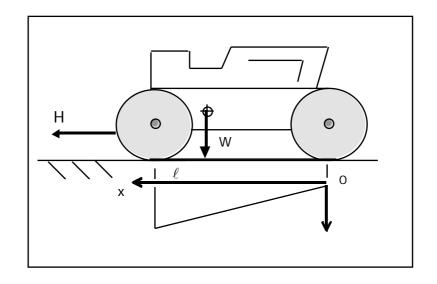
(i)   the track reaches maximum drawbar power at smaller slips on <u>rigid</u> stubble than on the <u>loose</u> cultivated soil;

(ii)   the <u>longer</u> track reaches maximum power at a (very much) smaller slip than does the <u>shorter</u> wheel (both on loose soil).
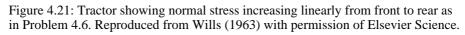
As another example Figure 4.20 shows the graph of drawbar power versus slip for a Deutz 2WD tractor tested on soil in three conditions (Hutchings, 1980). Again the drawbar power is reached at lower slips on the uncultivated (more rigid) soil than on the dry cultivated (loose) soil and both than on the soft, wet, cultivated soil.

The optimum slip (i') obtained from Figure 4.19 can be used, together with an appropriate rolling resistance , to calculate the maximum drawbar power.

$$Q_{max} = V_o (1 - i') \left[(Ac + W \tan \phi) \left(1 - \frac{k}{i'\ell} + \frac{k}{i'\ell} e^{-i'\ell/k} - R \right] \right. \qquad (4.24)$$

The setting up of the tractor to operate at this or other condition is discussed in Chapter 7.

Figure 4.21: Tractor showing normal stress increasing linearly from front to rear as in Problem 4.6. Reproduced from Wills (1963) with permission of Elsevier Science.

4.7 GENERAL PROBLEM

Problem 4.5 (Wills 1963)

Develop an expression for the tractive force - slip relationship developed by a track of width b, length ℓ and total weight W, operating on a frictional soil, if the normal stress increases linearly from zero at the front to a maximum at the rear as show in Figure 4.21.

Assume $\sigma = a\,x$ where a is a parameter to be determined. The requirement for vertical equilibrium is that:

$$W = \int_o^\ell a\,b\,x\,dx$$

$$= a\,b\,\left[\frac{x^2}{2}\right]_0^\ell$$

$$= a\,\frac{b\,\ell^2}{2}$$

Hence

$$a = \frac{2W}{b\,\ell^2}$$

$$\sigma = \frac{2W}{b\ell^2}\,x$$

$$s = \tan\phi\,(1- e^{-j/k})$$

$$\tau = \frac{2W}{b\ell^2}\,x\tan\phi\,(1- e^{-j/k})$$

From Section 4.4.4

$$H = \int_0^\ell b\,s\,dx$$

$$= \frac{2W}{\ell^2}\tan\phi\int_0^\ell x\,(1- e^{-j/k})\,dx$$

Substituting $j = ix$ and integrating gives

$$H = \frac{2W}{\ell^2}\tan\phi\,\left[\frac{x^2}{2} + \frac{xk}{i}\cdot e^{-ix/k} - \frac{k}{i}(-\frac{k}{i}\,e^{-ix/k})\right]_0^\ell$$

$$= W\tan\phi\,\left[1 - 2\,(\frac{k}{i\ell})^2\,(1 - e^{-i\ell/k} - \frac{i\ell}{k}\,e^{-i\ell/k}\right]$$

Problem 4.6

Plot the tractive force - slip graph for the track in Problem 4.5 using data from Problem 4.3. Compare the answers.

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